have maximum concentrative power (semi-vertical angle = 54° 44′). Under these circumstances the magnetic force at the vertex due to the free magnetism on the conical faces is—

$$18{,}930 \log_{10} \frac{b}{a},$$

where b is the diameter of the poles at the base of the cones, and a the diameter of the central neck.

The following are probable values of the intensity of magnetism when saturation is reached in the particular metals examined :—

	Saturation value of $\mathfrak{J}$.
Wrought iron	1700
Cast iron	1240
Nickel (with 0·75 per cent. of iron)	515
Nickel (with 0·56 per cent. of iron)	400
Cobalt (with 1·66 per cent. of iron)	1300

Experiments were also made with specimens of Vickers' tool steel, and other crucible steels, Whitworth's fluid-compressed steel, Bessemer steel, Siemens steel, and Hadfield's manganese steel. This last material, which is noted for its extraordinary impermeability to magnetic induction, was found to have a constant permeability of about 1·4 throughout the range of forces applied to it, namely, from 2000 to nearly 10,000 c.g.s.

The results are exhibited graphically by curves drawn in Rowland's manner to show the relation of the permeability to the magnetic induction. In the highest field examined, the permeability of wrought iron had fallen to about 2.

V. "The Waves on a rotating Liquid Spheroid of finite Ellipticity." By G. H. BRYAN, B.A. Communicated by Professor G. H. DARWIN. Received November 6, 1888.

(Abstract.)

The hydrodynamical problem of finding the waves or oscillations on a gravitating mass of liquid which when undisturbed is rotating as if rigid with finite angular velocity in the form of an ellipsoid or spheroid, was first successfully attacked by M. Poincaré in 1885.

In his important memoir "Sur l'Équilibre d'une Masse fluide animée d'un Mouvement de Rotation,"* Poincaré has (§ 13) obtained the differential equations for the oscillations of rotating liquid, and

* 'Acta Mathematica,' vol. 7.

shown that by a transformation of projection, the determination of the oscillations of any particular period is reducible to finding a suitable solution of Laplace's equation.

He then applies Lamé's functions to the case of the ellipsoid, showing that the differential equations are satisfied by a series of Lamé's functions referred to a certain auxiliary ellipsoid; the boundary conditions, however, involving ellipsoidal harmonics referred to both the auxiliary and actual fluid ellipsoids. At the same time, Poincaré's analysis does not appear to admit of any definite conclusions being formed as to the nature and frequencies of the various periodic free waves.

The present paper contains an application of Poincaré's methods to the simpler case when the fluid ellipsoid is one of revolution (Maclaurin's spheroid). The solution is effected by the use of the ordinary tesseral or zonal harmonics applicable to the fluid spheroid and the auxiliary spheroid required in solving the differential equation. The problem is thus freed from the difficulties attending the use of Lamé's functions, and is further simplified by the fact that each independent solution contains harmonics of only one particular degree and rank.

By substituting in the conditions to be satisfied at the surface of the spheroid, we arrive at a single boundary equation. If we are treating the forced tides due to a known periodic disturbing force, this equation determines their amplitude, and hence, the elevation of the tide above the mean surface of the spheroid at any point at any time. If there be no disturbing force it determines the frequencies of the various free waves determined by harmonics of given order and rank. Denoting by κ the ratio of the frequency of the free waves to twice the frequency of rotation of the liquid about its axis, the values of κ are the roots of a rational algebraic equation, and depend only on the eccentricity of the spheroid as well as the degree and rank of the harmonic, while the number of different free waves depends on the degree of the equation in κ. At any instant the height of the disturbance at any point of the surface is proportional to the corresponding surface harmonic on the spheroid multiplied by the central perpendicular on the tangent plane, and is of the same form for all waves determined by harmonics of any given degree and rank, whatever be their frequency, but the motions of the fluid particles in the interior will differ in nature in every case.

Taking first the case of zonal harmonics of the nth degree, we find that according as n is even or odd there will be $\frac{1}{2}n$ or $\frac{1}{2}(n+1)$, different periodic motions of the liquid. These are essentially oscillatory in character, and symmetrical about the axis of the spheroid. In all but one of these the value of κ is essentially less than unity, that is, the period is greater than the time of a semi-revolution of the liquid.

Taking next the tesseral harmonics of degree n and rank s, we find that they determine $n-s+2$ periodic small motions. These are essentially tidal waves rotating with various angular velocities about the axis of the spheroid, the angular velocities of those rotating in opposite directions being in general different. All but two of the values of κ are numerically less than unity, the periods of the corresponding tides at a point fixed relatively to the liquid being greater than the time of a semi-revolution of the mass.

The mean angular velocity of these $n-s+2$ waves is less than that of rotation of the mass by $2/\{s(n-s+2)\}$ of the latter.

In the two waves determined by any sectorial harmonic, the relative motion of the liquid particles is irrotational. The harmonics of degree 2 and rank 1 give rise to a kind of precession, of which there are two.

I have calculated the relative frequencies of several of the principal waves on a spheroid whose eccentricity is $1/\sqrt{2}$.

The question of stability is next dealt with, it being shown that in the present problem, in which the liquid forming the spheroid is supposed perfect, the criteria are entirely different from the conditions of secular stability obtained by Poincaré for the case when the liquid possesses any amount of viscosity, which latter depend on the energy being a minimum. In fact for a disturbance initially determined by any harmonic (provided that it is symmetrical with respect to the equatorial plane, since for unsymmetrical displacements the spheroid cannot be unstable), the limits of eccentricity consistent with stability are wider for a perfect liquid spheroid than for one possessing any viscosity. If we assume that the disturbed surface initially becomes ellipsoidal, the conditions of stability found by the methods of this paper agree with those of Riemann.

The case when the ellipticity, and therefore the angular velocity are very small is next discussed, it being shown that all but two of the waves, or all but one of the oscillations for any particular harmonic become unimportant, their periods increasing indefinitely.

In the case of those whose periods remain finite for a non-rotating spherical mass, the effect of a small angular velocity ω of the liquid is to cause them to turn round the axis with a velocity less than that of the liquid by ω/n.

Finally the methods of treating forced tides are further discussed.

The general cases of a "semi-diurnal" forced tide or of permanent deformations due to constant disturbing forces are mentioned in connexion with some peculiarities they present, and the paper concludes with examples of the determination of the forced tides due to the presence of an attracting mass, first when the latter moves in any orbit about the spheroid, secondly when it rotates uniformly about the spheroid in its equatorial plane.

The effects of such a body in destroying the equilibrium of the spheroid when the forced tide coincides with one of the free tides form the conclusion of this paper.

Presents, November 22, 1888.

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